Five Funny Bisimulations

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Standard Bisimulation (given variables P and agents A)

Syntax Language $\mathcal{L}(\Box)$: $\Box_a\varphi$ for 'φ is necessary (for agent a)'.

Structures Model $\mathcal{M} = (S, R, V)$ with pointed model $\mathcal{M}_s$.

Semantics $\mathcal{M}_s \models \Box_a\varphi$ iff $\mathcal{M}_t \models \varphi$ for all t such that $R_a st$.

Bisimulation Relation Z $(\neq \emptyset)$ between $\mathcal{M}$ and $\mathcal{M}'$ s.t. for all Zss':

atoms $s \in V(p)$ iff $s' \in V'(p)$;
forth $\forall t$: if $R_a st$, then $\exists t'$ such that $R'_a s' t'$ and Ztt';
back $\forall t'$: if $R'_a s' t'$, then $\exists t$ such that $R_a st$ and Ztt'.

Pointed models are bisimilar iff logically equivalent. (image-fin/sat)

$$\mathcal{M}_s \leftrightarrows \mathcal{M}'_{s'} \quad \text{iff} \quad \mathcal{M}_s \equiv \mathcal{M}'_{s'}$$

Example $s : \overline{p} \longrightarrow t : p \qquad u' : p \longleftarrow s' : \overline{p} \longrightarrow t' : p$

Contingency Bisimulation

Syntax Language $\mathcal{L}(\Delta)$

– $\Delta_a\varphi$ for 'φ is non-contingent' (φ is necessarily true or nec. false)
'agent a knows whether φ'

– $\nabla_a\varphi$ for 'φ is contingent' (φ can be both true and false)
'agent a is ignorant about φ'

Semantics

$\mathcal{M}_s \models \Delta_a\varphi$ iff $\forall t, u$ such that $R_ast, R_asu : \mathcal{M}_t \models \varphi$ iff $\mathcal{M}_u \models \varphi$

Example

Logically equivalent (but not all standard bisimilar) pointed models

$$\begin{tikzcd}
s : p \arrow[r] & t : p & s' : p \arrow[r] & t' : \overline{p} & s'' : p \arrow[r] \arrow[dr] & t'' : p \\
s''' : p & & & & & u'' : p
\end{tikzcd}$$

Contingency Bisimulation (single-agent, autobisimulation)

Contingency Bisimulation Relation Z ($\neq \emptyset$) on $\mathcal{M}$ s.t. for all Zss':

atoms	$s \in V(p)$ iff $s' \in V(p)$;
forth	if $\exists uv$ such that Rsu, Rsv, and not Zuv, then: $\forall t$: if Rst, then $\exists t'$ such that $Rs't'$ and Ztt';
back	if $\exists uv$ such that $Rs'u$, $Rs'v$, and not Zuv, then: $\forall t'$: if $Rs't'$, then $\exists t$ such that Rst and Ztt'.

Results

A standard bisimulation is a contingency bisimulation.
Pointed models are contingency bisimilar iff logically equivalent.
(On image-finite / saturated models, in the language $\mathcal{L}(\Delta)$.)
Contingency logic is less expressive than necessity logic.
Almost definability $\nabla\psi \to (\Box\varphi \leftrightarrow \Delta\varphi \wedge \Delta(\psi \to \varphi))$ is valid.

Contingency Bisimulation — Example

Logically equivalent and contingency bisimilar

$$
\begin{tikzcd}
s : p \arrow[r] & t : p & s' : p \arrow[r] & t' : \overline{p} & s'' : p \arrow[r] \arrow[dr] & t'' : p \\
s''' : p & & & & & u'' : p
\end{tikzcd}
$$

Also logically equivalent and contingency bisimilar

$$
\begin{tikzcd}
& s : p \arrow[dl] \arrow[d] \arrow[dr] & & & s' : p \arrow[d] \arrow[dr] & \\
t : p & u : p & v : \overline{p} & & u' : p & v' : \overline{p}
\end{tikzcd}
$$

Contingency Bisimulation Contraction

Given a model

$$\begin{tikzcd} s : p \arrow[d] & u : p \arrow[d] \\ t : p & v : \overline{p} \end{tikzcd}$$

This is not the contraction

$$\begin{tikzcd} {[s]_Z : p} \arrow[loop left] \arrow[d] \\ {[v]_Z : \overline{p}} \end{tikzcd}$$

But this is

$$\begin{tikzcd} {[s]_Z : p} \\ {[v]_Z : \overline{p}} \end{tikzcd}$$

Where $[s]_Z = \{s, t, u\}$ and $[v]_Z = \{v\}$ (and Z is the maximal bisimulation).

Contingency bisimulation contraction $[\mathcal{M}] = ([S], [R], [V])$ def. as

- $[S] = \{[s]_Z \mid s \in S\}$ where $[s]_Z = \{t \in S \mid Zst\}$ (Z is maximal);
- $[R][s][t]$ iff $\exists s't' : Zss', Ztt'$, and $Rs't'$, and
  $\exists uv : Rs'u, Rs'v$, and not Zuv;
- $[V](p) = \{[s]_Z \mid s \in V(p)\}$.

By taking the reflexivity closure of the relation $[R]$, the bisimulation contraction of an $S5$ model is an $S5$ model.

Contingency Bisimulation — Pubs and People

Jie Yanjing

Jie Fan, Yanjing Wang, Hans vD: *Almost Necessary*. Advances in Modal Logic 2014: 178–196.

Jie Fan expects to defend his PhD in 2015 at Peking University.

Awareness Bisimulation

Hans is uncertain if there is coffee (p).

$$
\begin{tikzcd}
s : \overline{p} \arrow[loop left] \arrow[r, leftrightarrow] & t : p \arrow[loop right]
\end{tikzcd}
$$

Tim informs Hans that coffee and orange juice (q) are not both served.

$$
\begin{tikzcd}
u : \overline{p}q \arrow[loop left] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\
s : \overline{pq} \arrow[loop left] \arrow[r, leftrightarrow] & t : p\overline{q} \arrow[loop right]
\end{tikzcd}
$$

Awareness Bisimulation

Hans is uncertain if there is coffee (p).

$$\begin{tikzcd} s : \overline{p} \arrow[loop left] \arrow[r, leftrightarrow] & t : p \arrow[loop right] \end{tikzcd}$$

Tim informs Hans that coffee and orange juice (q) are not both served.

$$\begin{tikzcd} u : \overline{p}q \arrow[loop left] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\ s : \overline{pq} \arrow[loop left] \arrow[r, leftrightarrow] & t : p\overline{q} \arrow[loop right] \end{tikzcd}$$

$$\begin{tikzcd} u : \overline{p}{\color{red}q} \arrow[loop left] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\ s : \overline{p{\color{red}q}} \arrow[loop left] \arrow[r, leftrightarrow] & t : p{\color{red}\overline{q}} \arrow[loop right] \end{tikzcd}$$

The model before Hans was informed.

Awareness models and explicit knowledge

Syntax $\Box_a\varphi$ for 'agent a implicitly knows φ'
$K_a^E\varphi$ for 'agent a explicitly knows φ'
$A_a\varphi$ for 'agent a is aware of φ'

Structures *Awareness model* $(S, R, \mathcal{A}, V)$ with *awareness function* $\mathcal{A}$ assigning to each state and agent the variables it is aware of.

Semantics $\mathcal{M}_s \models A_a\varphi$ iff $v(\varphi) \subseteq \mathcal{A}_a(s)$
$\mathcal{M}_s \models K_a^E\varphi$ iff $\mathcal{M}_s \models \Box_a\varphi \wedge A_a\varphi$

Example Bisimilar 'for the agent' but not modally equivalent.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

We have that $s \models K^E\Box p$ but $s' \not\models K^E\Box p$.
In states t and t', the agent is unaware of p, thus indifferent to the different value of p in u and u'. We want s and s' to be bisimilar...

Awareness bisimulation

Let $Q \subseteq P$. A Q awareness bisimulation is a collection of binary relations $Z_{Q'}$ between $\mathcal{M}$ and $\mathcal{M}'$ for all $Q' \subseteq Q$ s.t. for all $Z_{Q'}ss'$:

atoms	$s \in V(p)$ iff $s' \in V'(p)$;
aware	$\mathcal{A}_a(s) \cap Q' = \mathcal{A}'_a(s') \cap Q'$;
forth	$\forall t$: if $R_a st$ then $\exists t'$ such that $R'_a s' t'$ and $Z_{Q' \cap \mathcal{A}_a(s)} tt'$;
back	$\forall t'$: if $R'_a s' t'$ then $\exists t$ such that $R_a st$ and $Z_{Q' \cap \mathcal{A}'_a(s')} tt'$.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

Example of a p awareness bisimulation:

$$\begin{array}{rcl} Z_p & = & \{(s, s'), (t, t')\} \\ Z_\emptyset & = & \{(u, u')\} \end{array}$$

Another (maximal) awareness bisimulation, with $Z'_\emptyset \subseteq Z'_p$.

$$\begin{array}{rcl} Z'_p & = & \{(s, s'), (t, t')\} \\ Z'_\emptyset & = & \{(s, s'), (t, t'), (u, u')\} \end{array}$$

Awareness bisimulation and dynamics — Example

Initial models, as before. Awareness bisimilar, and modally equivalent in $\mathcal{L}(K^E)$.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

The agent becomes aware of p.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

Clearly the models no longer awareness bisimilar, and $K^E K^E p$ is now a distinguishing formula. Dynamics increases expressivity.

Results — Awareness Logics $\mathcal{L}(A, K^E)$, $\mathcal{L}(A, K^S)$, $\mathcal{L}(A, \Box)$

Speculative knowledge — a novel epistemic operator

$\mathcal{M}_s \models K_a^S \varphi$ iff $\mathcal{M}'_{t'} \models \varphi$ for all t, t' s.t. $R_a st$ and $\mathcal{M}_t \leftrightarroweq_{\mathcal{A}_a(s)} \mathcal{M}'_{t'}$

Explicit $\Rightarrow$ speculative $\Rightarrow$ implicit: $K_a^E \varphi \to K_a^S \varphi$ and $K_a^S \varphi \to \Box_a \varphi$.

K^E: Awareness bisimilarity corresponds to logical equivalence.
K^S: Awareness bisimilarity corresponds to logical equivalence.
$\Box$: Standard bisimilarity corresponds to logical equivalence.

The logics of explicit knowledge and speculative knowledge are equally expressive. The logic of implicit knowledge is more expressive. Adding dynamics makes all 3 logics equally expressive.

Awareness Bisimulation — Pubs and People

Tim

Fer

Yi

Hans vD, Tim French, Fernando Velázquez Quesada, Yi N. Wang: *Knowledge, Awareness, and Bisimulation*, Proc. of TARK 2013.

With work unrelated to bisimulation:
Fernando obtained his PhD in 2011 at University of Amsterdam.
Yi obtained his (2nd) PhD in 2013 at University of Bergen.

Plausibility Bisimulation — Example

Plausibility models: equivalence classes encode knowledge, where in each equivalence class the states are ordered into more and less *plausible* states. If s is at least as plausible as t, we write $t \geq s$. (In the picture: an arrow from t to s. We assume reflexive closure.)

$$\begin{tikzcd} w_1 : p & w_2 : \overline{p} \arrow[l] & w_3 : p \arrow[l] \arrow[ll, bend left] \end{tikzcd} \qquad \begin{tikzcd} v_1 : p & v_2 : \overline{p} \arrow[l] \end{tikzcd}$$

- $K\varphi$: You *know* φ iff φ is true in all possible states.
- $B\varphi$: You *believe* φ iff φ is true in the most plausible states.
- $B^{\psi}\varphi$: You *conditionally believe* φ iff φ is true in the most plausible states satisfying the condition (ψ).
- $\Box\varphi$: You *safely believe* φ iff φ is true cond. to any true restr.

Example $\quad w_1 \models Bp$ but $w_1 \not\models Kp$. $\quad w_1 \models \Box p$ but $w_3 \not\models \Box p$.
The models are logically equivalent in the logics of conditional belief and knowledge. They are not standard bisimilar. A notion of plausibility bisimulation makes them bisimilar. With another semantics for safe belief, they are also logically equiv. in that logic.

Multi-agent example of plausibility bisimilar models

Single-agent: we make models *plausibility bisimilar* by identifying states with the same valuation (with the *most* plausible state).

$$\begin{tikzcd} w_1 : p & w_2 : \overline{p} \arrow[l] & w_3 : p \arrow[l] \arrow[ll, bend left] \end{tikzcd} \qquad \begin{tikzcd} v_1 : p & v_2 : \overline{p} \arrow[l] \end{tikzcd}$$

But in the multi-agent case this no longer works. For example:

$$\begin{tikzcd} w_1 : p \arrow[d, "b"'] & w_2 : p \arrow[l, "a"'] & w_3 : p \arrow[l, "a"'] \arrow[ll, bend left, "a"] \arrow[d, "b"'] \\ w_4 : \overline{p} & & w_5 : \overline{p} \end{tikzcd} \qquad \begin{tikzcd} v_1 : p \arrow[d, "b"'] & v_2 : p \arrow[l, "a"'] \\ v_3 : \overline{p} & \end{tikzcd}$$

In *plausibility bisimulation* the back and forth clauses refer to the bisimulation in the condition (similar to *contingency bisimulation*).
[forth] clause for Zss': if $s \geq_a^Z t$, $\exists t'$ such that $s' \geq_a^Z t'$ and Ztt';

Results

Results

Read the PhD theses of Martin and Mikkel!

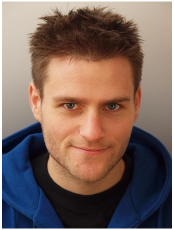
Or wait for this to be published in a journal or available on ArXiV.

Plausibility Bisimulation — Pubs and People

Mikkel

Thomas

Martin



Mikkel Birkegaard Andersen, Thomas Bolander, Hans vD, Martin Holm Jensen: *Bisimulation for Single-Agent Plausibility Models*. Australasian AI 2013: 277-288.

Martin obtained his PhD in 2014 at Technical University Denmark.
Mikkel will defend his PhD in 2014 at Technical Univ. Denmark.

Refinement

Given this model $\mathcal{M}$

$$\circ \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}$$

It is (standard) bisimilar to the 'blown up' model

$$\bullet \longleftarrow \bullet \longleftarrow \bullet \longleftarrow \circ \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}'$$

A more radical structural transformation is a submodel like

$$\circ \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}''$$

Now consider this: neither a bisimilar copy nor a model restriction.

$$\bullet \longleftarrow \circ \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}'''$$

$\mathcal{M}'''$ is a *refinement* of $\mathcal{M}$: a model restriction of a bisimilar copy.

Refinement — a refinement satisfies back but not forth

A *B refinement* (linking M_s & $M'_{s'}$, notation $M_s \succeq_B M'_{s'}$, where $B \subseteq A$) is a relation $Z_B \subseteq S \times S'$ (containing (s, s')) that satisfies:
— 'atoms'
— 'back' for all agents $a \in B$
— 'forth' and 'back' for all agents $a \in A \setminus B$

Consider again $\mathcal{M}$ and $\mathcal{M}'''$. Then $\mathcal{M}_1 \succeq \mathcal{M}'''_6$. (Unlabeled.)
The refinement relation is $Z = \{(1,6),(2,5),(2,7),(3,8)\}$.

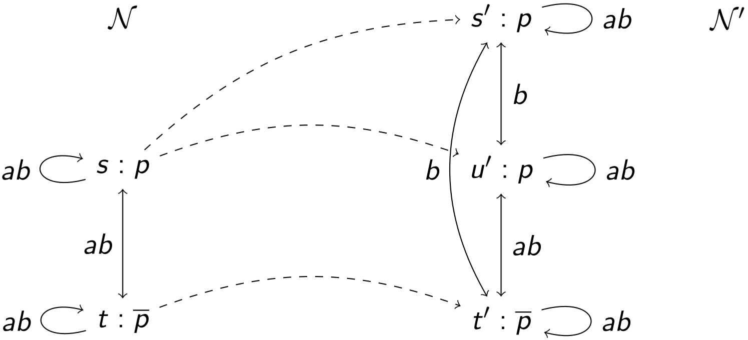
$$\begin{tikzcd}
 & 1\circ \arrow[r] \arrow[d, dashed] & 2\bullet \arrow[r] \arrow[d, dashed] \arrow[dll, dashed] & 3\bullet \arrow[r] \arrow[d, dashed] & 4\bullet & \mathcal{M} \\
5\bullet & 6\circ \arrow[l] \arrow[r] & 7\bullet \arrow[r] & 8\bullet & & \mathcal{M}'''
\end{tikzcd}$$

$$\mathcal{M}_s \models \forall_a \varphi \text{ iff for all } \mathcal{M}'_{s'} : \mathcal{M}_s \succeq_a \mathcal{M}'_{s'} \text{ implies } \mathcal{M}'_{s'} \models \varphi$$

$\forall_a \varphi$ is true in a pointed model iff φ is true in all its *a-refinements*.

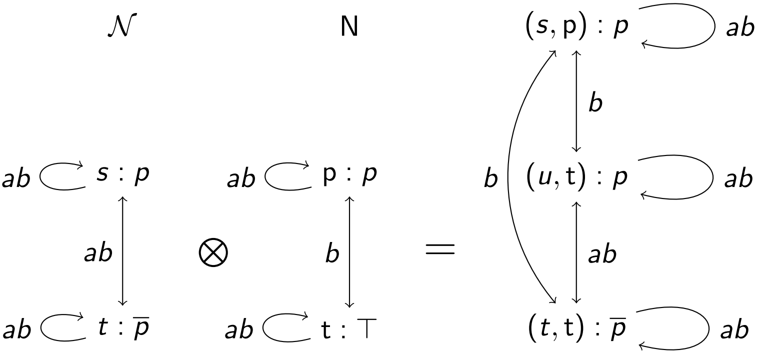
Refinement Modal Logic

Action model execution is a refinement, and vice versa.
Consider $\mathcal{N} \succeq_a \mathcal{N}'$ below.



Refinement Modal Logic

The previous slide depicted $\mathcal{N} \succeq_a \mathcal{N}'$.
Same models, but $\mathcal{N}'$ as $\mathcal{N} \otimes \mathsf{N}$, where N is an action (model).



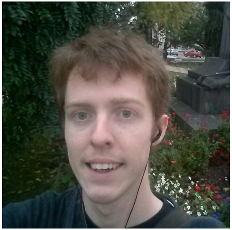
Results for Refinement Modal Logic

- Action model execution is a refinement, and, on finite models, every refinement is the execution of an action model.
- Axiomatization is more elegant if you employ the coalgebraic cover modality instead of the necessity / possibility modalities. $\nabla\{\varphi, \psi\}$ is defined as $\Diamond\varphi \wedge \Diamond\psi \wedge \Box(\varphi \vee \psi)$.
- Refinement modal logic has a complete axiomatization and is equally expressive as multi-agent modal logic.
- Refinement is bisimulation plus model restriction, and refinement quantification is bisimulation quantification followed by relativization: $\exists\varphi$ is equivalent to $\tilde{\exists} q \varphi^q$.
- Refinement epistemic logic (on $S5$ models) has a complete axiomatization.
- Refinement μ calculus is also axiomatized.
  (Future suspects: refinement CTL, refinement PDL, ...)

Refinement — Pubs and People

Laura

James



Sophie

Laura Bozzelli, Hans vD, Tim French, James Hales, Sophie Pinchinat: *Refinement Modal Logic*. Information and Computation 239 (2014) 303–339.

James expects to defend his PhD in 2015 at U o Western Australia.

Bisimulation for Sabotage

Sabotage logic was proposed by Johan van Benthem. A traveller tries to get from A to B by train. The railway operator sabotages (removes links from) the network. It contains an operator for what is true after one removes a pair from the accessibility relation.

$$\mathcal{M}_s \models \langle \mathsf{sb} \rangle \varphi \quad \text{iff} \quad \text{there are } t, u \in S \text{ such that } \mathcal{M}_s^{-tu} \models \varphi$$

where $\mathcal{M}^{-tu}$ is as $\mathcal{M} = (S, R, V)$ except that $R^{-tu} = R \setminus \{(t, u)\}$. The sabotage operation sb is not bisimulation preserving.

$$\mathcal{M}: \begin{tikzcd} s : p \arrow[r, bend left] & t : p \arrow[l, bend left] \end{tikzcd} \qquad \mathcal{M}': \begin{tikzcd} s' : p \arrow[loop left] \end{tikzcd}$$

We have $\mathcal{M}_s \underline{\leftrightarrow} \mathcal{M}'_{s'}$. But $\mathcal{M}_s \not\models [\mathsf{sb}]\Box\bot$ whereas $\mathcal{M}'_{s'} \models [\mathsf{sb}]\Box\bot$. Correspondence can be regained by strengthening the requirements of bisimulation. Instead of a *standard bisimulation* Z as a relation between states, containing pair (s, s'), a *sabotage bisimulation* is a *relation between state-relation pairs* containing $((s, R), (s', R'))$. Also, we have to add clauses for the dynamic sabotage modality.

Bisimulation for sabotage logic — Pubs and People

Carlos

Raul

Carlos Areces, Raul Fervari, Guillaume Hoffmann: *Moving Arrows and Four Model Checking Results*. WoLLIC 2012: 142–153.
Carlos Areces, Hans vD, Raul Fervari, François Schwarzentruber: *Logics with Copy and Remove*. WoLLIC 2014: 51–65.

Raul Fervari obtained his PhD in 2014 at Univ. of Córdoba (Arg.).