Interrogative dependencies and the constructive content of inquisitive proofs

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TU Delft — 12th May 2014

Three natural logical notions

- Standard entailment
  $\neg p \quad \models \quad \neg(p \wedge q)$
- Resolution
  $\neg p \quad \searrow \quad ?(p \wedge q)$
- Interrogative dependency
  $p \leftrightarrow q \wedge r, \;\; ?q, \;\; ?r \quad \rightrightarrows \quad ?p$

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  $\neg p \quad \models \quad \neg(p \wedge q)$
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Goal of this talk

Show that these notions are all instances of logical consequence in a uniform logic of information and issues.

Overview

Part I: semantics

Inquisitive semantics for declaratives and interrogatives.

Part II: entailment

Declarative entailment, resolution and interrogative dependencies.

Part III: logical calculus

Computational content of inquisitive proofs.

Part I

Semantics

Definition (Syntax of InqD_π)

$\mathcal{L}$ consists of a set $\mathcal{L}_!$ of declaratives and a set $\mathcal{L}_?$ of interrogatives.

$$\mathcal{L}_! \quad \alpha \quad ::= \quad p \quad | \quad \bot \quad | \quad \alpha \wedge \alpha \quad | \quad \varphi \to \alpha$$

$$\mathcal{L}_? \quad \mu \quad ::= \quad ?\{\alpha_1, \dots, \alpha_n\} \quad | \quad \mu \wedge \mu \quad | \quad \varphi \to \mu$$

$$\mathcal{L} \quad \varphi \quad ::= \quad \alpha \quad | \quad \mu$$

Abbreviations

- If $\alpha \in \mathcal{L}_!$, $\quad \neg\alpha := \alpha \to \bot$
- If $\alpha, \beta \in \mathcal{L}_!$, $\quad \alpha \vee \beta := \neg(\neg\alpha \wedge \neg\beta)$
- If $\alpha \in \mathcal{L}_!$, $\quad ?\alpha := ?\{\alpha, \neg\alpha\}$

Notational convention on meta-variables

	Declaratives	Interrogatives	Full language
Formulas	α, β, γ	μ, ν, λ	φ, ψ, χ
Sets of formulas	Γ	Λ	Φ

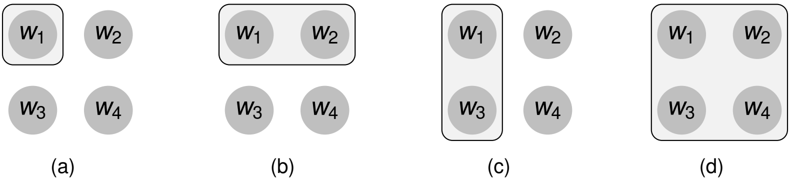
Definition (Models)

A model for a set $\mathcal{P}$ of atoms is a pair $M = \langle \mathcal{W}, V \rangle$ where:

- $\mathcal{W}$ is a set, whose elements we call possible worlds
- $V : \mathcal{W} \to \wp(\mathcal{P})$ is a valuation function

Definition (Information states)

An information state is a set of possible worlds.



Semantics

- Usually, semantics assigns truth-conditions at worlds.
- However, our language now contains interrogatives as well.
- Interrogative meaning = resolution conditions at states.
- Our semantics is defined by a relation $s \models \varphi$ of support between information states s and formulas φ, where:

  Declaratives: $s \models \alpha \iff \alpha$ is established in s

  Interrogatives: $s \models \mu \iff \mu$ is resolved in s

Definition (Support)

1. $s \models p \iff p \in V(w)$ for all worlds $w \in s$
2. $s \models \bot \iff s = \emptyset$
3. $s \models ?\{\alpha_1, \ldots, \alpha_n\} \iff s \models \alpha_1$ or $\ldots$ or $s \models \alpha_n$
4. $s \models \varphi \wedge \psi \iff s \models \varphi$ and $s \models \psi$
5. $s \models \varphi \to \psi \iff$ for any $t \subseteq s$, if $t \models \varphi$ then $t \models \psi$

Fact

- Persistence: if $s \models \varphi$ and $t \subseteq s$, then $t \models \varphi$
- Absurd state: $\emptyset \models \varphi$ for any φ

Definition (Truth)

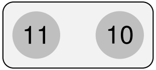
$M, w \models \varphi \overset{def}{\iff} M, \{w\} \models \varphi$

Fact (Truth-conditions)

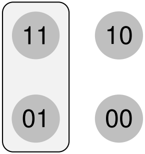
- $M, w \models p \iff p \in V(w)$
- $M, w \not\models \bot$
- $M, w \models ?\{\alpha_1, \ldots, \alpha_n\} \iff M, w \models \alpha_1$ or $\ldots$ or $M, w \models \alpha_n$
- $M, w \models \varphi \wedge \psi \iff M, w \models \varphi$ and $M, w \models \psi$
- $M, w \models \varphi \rightarrow \psi \iff M, w \not\models \varphi$ or $M, w \models \psi$

Fact (Declaratives are truth-conditional)

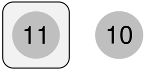
$M, s \models \alpha \iff M, w \models \alpha$ for all $w \in s$



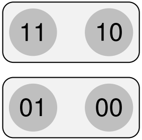
p



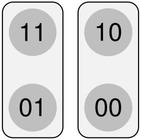
q



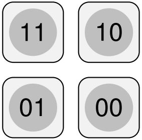
$p \wedge q$



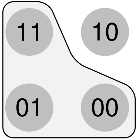
$?p$



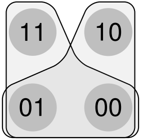
$?q$



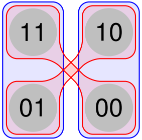
$?p \wedge ?q$



$p \rightarrow q$



$p \rightarrow ?q$



$?p \rightarrow ?q$

Part II

Entailment

Definition (Resolutions)

- $\mathcal{R}(\alpha) = \{\alpha\}$ if α is a declarative
- $\mathcal{R}(?\{\alpha_1, \ldots, \alpha_n\}) = \{\alpha_1, \ldots, \alpha_n\}$
- $\mathcal{R}(\mu \wedge \nu) = \{\, \alpha \wedge \beta \mid \alpha \in \mathcal{R}(\mu)$ and $\beta \in \mathcal{R}(\nu)\}$
- $\mathcal{R}(\varphi \to \mu) = \{\, \bigwedge_{\alpha \in \mathcal{R}(\varphi)} \alpha \to f(\alpha) \mid f : \mathcal{R}(\varphi) \to \mathcal{R}(\mu)\}$

Fact

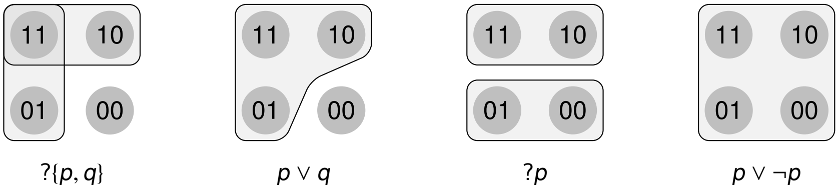
$M, s \models \varphi \iff M, s \models \alpha$ for some $\alpha \in \mathcal{R}(\varphi)$

Definition (Presupposition of an interrogative)

$\pi_\mu := \bigvee \mathcal{R}(\mu)$

Fact

$M, w \models \mu \iff M, w \models \pi_\mu$



Definition (Resolutions of a set)

A resolution of a set Φ is a set of declaratives Γ such that:

- $\forall\varphi \in \Phi \;\; \exists\alpha \in \Gamma$ s.t. $\alpha \in \mathcal{R}(\varphi)$
- $\forall\alpha \in \Gamma \;\; \exists\varphi \in \Phi$ s.t. $\alpha \in \mathcal{R}(\varphi)$

The set of resolutions of Φ is denoted $\mathcal{R}(\Phi)$.

Example

The following are resolutions of $\{p, ?q, ?r\}$:

- $\{p, q, r\}$
- $\{p, q, \neg r\}$
- $\{p, q, \neg q, r\}$
- $\ldots$

Definition (Entailment)

$\Phi \models \psi \iff$ for all M, s, if $M, s \models \Phi$ then $M, s \models \psi$

We will distinguish two cases:

- the conclusion ψ is declarative
- the conclusion ψ is interrogative

Fact (Entailment towards declaratives is truth-conditional)

$\Phi \models \alpha \iff$ for all M, w, if $M, w \models \Phi$ then $M, w \models \alpha$

Corollary (Conservativity on classical logic)

If Γ, α are propositional formulas, $\Gamma \models \alpha \iff \Gamma \models_{\mathsf{CPL}} \alpha$

Corollary

$\Gamma, \Lambda \models \alpha \iff \Gamma, \Pi_\Lambda \models \alpha$ where $\Pi_\Lambda = \{\pi_\mu \mid \mu \in \Lambda\}$

Fact

$\Phi \models \psi \iff \forall \Gamma \in \mathcal{R}(\Phi) \quad \exists \alpha \in \mathcal{R}(\psi) \quad \text{s.t.} \quad \Gamma \models \alpha$

Entailment towards an interrogative

$\Gamma, \Lambda \models \mu \iff$ given Γ, any resolution of Λ entails a resolution of μ

$\iff$ given Γ, Λ determines μ

Example

$p \leftrightarrow q \wedge r, \quad ?q \wedge ?r \quad \models \quad ?p$

- $p \leftrightarrow q \wedge r, \quad q \wedge r \quad \models \quad p$
- $p \leftrightarrow q \wedge r, \quad q \wedge \neg r \quad \models \quad \neg p$
- $p \leftrightarrow q \wedge r, \quad \neg q \wedge r \quad \models \quad \neg p$
- $p \leftrightarrow q \wedge r, \quad \neg q \wedge \neg r \quad \models \quad \neg p$

- Interrogative dependencies are instances of entailment with
  - interrogative conclusion
  - some interrogative assumptions

- Such dependencies are internalized in the language as implications. E.g.: $?p \to ?q$

- The particular dependency between two interrogatives may itself be one of the variables at stake. Consider:

$$?p, \quad ?p \to ?q \quad \models \quad ?q$$

Corollary (Split)

$\Gamma \models \mu \iff \Gamma \models \alpha$ for some $\alpha \in \mathcal{R}(\mu)$

Declarative-to-interrogative entailment

$\Gamma \models \mu \iff \Gamma$ resolves μ.

Example

$\neg p \models ?(p \wedge q)$

Thus, inquisitive entailment encompasses:

- Declarative entailment:
  $\Gamma \models \alpha \iff \Gamma$ implies α

- Resolution:
  $\Gamma \models \mu \iff \Gamma$ resolves μ

- Interrogative dependency:
  $\Gamma, \Lambda \models \mu \iff$ given Γ, Λ determines μ

Part III

Logical calculus

Conjunction

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$$

Implication

$$\frac{\begin{matrix}[\alpha] \\ \vdots \\ \beta\end{matrix}}{\alpha \to \beta} \qquad \frac{\alpha \quad \alpha \to \beta}{\beta}$$

Falsum

$$- \qquad \frac{\bot}{\alpha}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{array}{c}[\varphi] \\ \vdots \\ \psi\end{array}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$- \qquad \frac{\bot}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{array}{c}[\varphi]\\ \vdots \\ \psi\end{array}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$- \qquad \frac{\bot}{\varphi}$$

Interrogative

$$\frac{\alpha_i}{?\{\alpha_1, \dots, \alpha_n\}} \qquad \frac{\begin{array}{c}[\alpha_1]\\ \vdots \\ \varphi\end{array} \quad \dots \quad \begin{array}{c}[\alpha_n]\\ \vdots \\ \varphi\end{array} \quad ?\{\alpha_1, \dots, \alpha_n\}}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Conjunction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

Implication

$$\frac{\begin{array}{c}[\varphi] \\ \vdots \\ \psi\end{array}}{\varphi \to \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi}$$

Falsum

$$- \qquad \frac{\bot}{\varphi}$$

Interrogative

$$\frac{\alpha_i}{?\{\alpha_1, \dots, \alpha_n\}} \qquad \frac{\begin{array}{c}[\alpha_1] \\ \vdots \\ \varphi\end{array} \quad \dots \quad \begin{array}{c}[\alpha_n] \\ \vdots \\ \varphi\end{array} \quad ?\{\alpha_1, \dots, \alpha_n\}}{\varphi}$$

Double negation axiom

$$\neg\neg\alpha \to \alpha$$

Kreisel-Putnam axiom

$$(\alpha \to ?\{\beta_1, \dots, \beta_m\}) \ \to \ ?\{\alpha \to \beta_1, \dots, \alpha \to \beta_m\}$$

Theorem (Resolution theorem)

$$\Phi \vdash \psi \quad \iff \quad \forall \Gamma \in \mathcal{R}(\Phi) \quad \exists \alpha \in \mathcal{R}(\psi) \quad \text{s.t.} \quad \Gamma \vdash \alpha$$

Constructive significance

The left-to-right proof describes a procedure that given a proof $P : \Phi \vdash \psi$

- takes a specific resolution Γ of Φ
- by induction on P, constructs a proof $\Theta(P, \Gamma) : \Gamma \vdash \alpha$ concluding with some specific resolution α of ψ

Thus, P can be seen as encoding a specific dependency of ψ on Φ or, a method for turning resolutions of Φ into resolutions of ψ.

Example

- Consider the proof
  $P \;:\; p \leftrightarrow q \wedge r,\;\; ?q \wedge ?r \;\vdash\; ?p$

$$
\dfrac{
\dfrac{?q \wedge ?r}{?q}\,(\wedge e)
\qquad
\dfrac{
\dfrac{?q \wedge ?r}{?r}\,(\wedge e)
\qquad
\dfrac{\dfrac{[q] \quad [r] \quad p \leftrightarrow q \wedge r}{p}\,(P_1)}{?p}\,(?i)
\qquad
\dfrac{\dfrac{[\neg r] \quad p \leftrightarrow q \wedge r}{\neg p}\,(P_2)}{?p}\,(?i)
}{?p}\,(?e)
\qquad
\dfrac{\dfrac{[\neg q] \quad p \leftrightarrow q \wedge r}{\neg p}\,(P_3)}{?p}\,(?i)
}{?p}\,(?e)
$$

- Suppose $?q \wedge ?r$ is resolved to $q \wedge r$.

- The procedure delivers the proof
  $\Theta(P, q \wedge r) \;:\; p \leftrightarrow q \wedge r,\;\; q \wedge r \;\vdash\; p$

$$
\dfrac{\dfrac{q \wedge r}{q}\,(\wedge e) \qquad \dfrac{q \wedge r}{r}\,(\wedge e) \qquad p \leftrightarrow q \wedge r}{p}\,(P_1)
$$

Example

- Consider the proof
  $P \;:\; p \leftrightarrow q \wedge r,\;\; ?q \wedge ?r \;\vdash\; ?p$

$$\dfrac{\dfrac{?q\wedge ?r}{?q}\,(\wedge e)\qquad \dfrac{\dfrac{?q\wedge ?r}{?r}\,(\wedge e)\qquad \dfrac{\dfrac{[q]\quad [r]\quad p\leftrightarrow q\wedge r}{p}\,(P_1)}{?p}\,(?i)\qquad \dfrac{\dfrac{[\neg r]\quad p\leftrightarrow q\wedge r}{\neg p}\,(P_2)}{?p}\,(?i)}{?p}\,(?e)\qquad \dfrac{\dfrac{[\neg q]\quad p\leftrightarrow q\wedge r}{\neg p}\,(P_3)}{?p}\,(?i)}{?p}\,(?e)$$

- Suppose $?q \wedge ?r$ is resolved to $q \wedge \neg r$.

- The procedure delivers the proof
  $\Theta(P, q \wedge \neg r) \;:\; p \leftrightarrow q \wedge r,\;\; q \wedge \neg r \;\vdash\; \neg p$

$$\dfrac{\dfrac{q\wedge \neg r}{\neg r}\,(\wedge e)\qquad p\leftrightarrow q\wedge r}{\neg p}\,(P_2)$$

Wrapping up

Semantics

We can generalize classical logic beyond truth-conditions, to a setting where we can treat both information and issues.

Entailment

This setting yields a general entailment relation, encompassing declarative entailment, resolution, and interrogative dependency.

Logical calculus

The associated logic is a conservative extension of classical logic, with constructive features when it comes to interrogatives.

Proofs

Proofs involving interrogatives have a specific computational content: they encode methods for turning resolutions of the interrogative assumptions into resolutions of the interrogative conclusion.


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
BÏYAN
SHUKRIA
TINGKI
YAQHANYELAY
TASHAKKUR ATU
SUKSAMA
EKHMET
MEHRBANI
GRAZIE
MAAKE
KOMAPSUMNIDA
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
PALDIES
BOLZÏN
MERCI

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