Bayesian Decision Theory

The Bayesian Calculi

- Plausibility Judgments → Bayesian Probability Theory
  – Laplace (1774)

- Relevancy Judgments → Bayesian Information Theory
  – See introduction (Knuth, 2014):

    Information-Based Physics: an observer-centric foundation

- Decision Making → Bayesian Decision Theory [?]
  – van Erp, Linger, & van Gelder (2014)

The Bayesian Decision Theory

1. Use the product and sum rule of Bayesian probability theory to construct outcome probability distributions.

2. If our outcomes are monetary in nature, then by way of the Bernoulli law we may map utilities to the monetary outcomes of our outcome probability distributions.

3. Maximize either the lower or upper bounds, depending on the specific context of the problem of choice we are studying, of the resulting utility probability distribution.

A Comparison

- Expected utility theory (Bernoulli, 1738):
  - criterion for action:
    - Compare the **means** of the utility probability distributions.
- Bayesian decision theory (2014):
  - criterion for action:
    - Compare the **upper and lower bounds** of the utility probability distributions.

Degree of Freedom

1. The product and sum rule of the Bayesian probability theory for the construction of outcome probability distributions are dictated by the desideratum of consistency.
   - (Cox 1946, Jaynes 2003, Knuth & Skilling 2010).

2. **Bernoulli law only degree of freedom.**

3. Positions of utility probability distributions are to be compared by some function of the cumulants of the utility probability distributions.
   - More-to-the-Right = More Profitable/Less Disadvantageous
   - Unity function of the first cumulant (*e.g.* Expected Utility Theory) leads to Ellsberg and Allais paradoxes. CI-bound functions of the cumulants solve for these paradoxes trivially.

Bernoulli law/Weber-Fechner law/ Steven's Power law/Thurnstone's Satisfaction law

- A lattice theoretical derivation of the Bernoulli law is given in:

  Fact Sheet Research on Bayesian Decision Theory

  (van Erp, Linger, & van Gelder, 2014)

- The product and sum rule of the **probability calculus** derived as a **quantification**, by way of **consistency constraints**, on the **lattice of statements**.

- The product and sum rule of the **inquiry calculus** derived as a **quantification** , by way of **consistency constraints**, on the **lattice of questions**.

- The **Bernoulli law** derived as a **quantification** , by way of **consistency constraints**, on the **lattice of ordering**.

- We have send the fact sheet to **Kevin Knuth** for an **informal peer review** of our derivation of the Bernoulli law.

Assigning Utilities

- The Weber-Fechner law is a law of experimental psychology.

- The Weber-Fechner law governs the perception of increments in stimuli.

- Bernoulli law = Weber-Fechner law

The Weber-Fechner law

- initial stimulus strength: S
- increment stimulus strength: ΔS
- increment in subjective intensity: $u\left(\Delta S \mid S_0\right) = q \log \frac{S + \Delta S}{S}$

Controversy?

The Weber-Fechner law:

$$u = q \log \frac{S + \Delta S}{S}$$

Steven's Power law:

$$v = \left(\frac{S + \Delta S}{S} \right)^q \quad \rightarrow \quad \log(v) = q \log \frac{S + \Delta S}{S} = u$$

Assumption

- If monies are stimuli that move us, then we may use the Bernoulli/Weber-Fechner law to model the intensity of monetary increments.

- Utilities = Intensity of monetary increments

- Assign utilities by way of Bernoulli/Weber-Fechner law.

Kahneman and Tversky

- Loss Aversion
  - The psychological phenomenon that losses way heavier than equal gains.

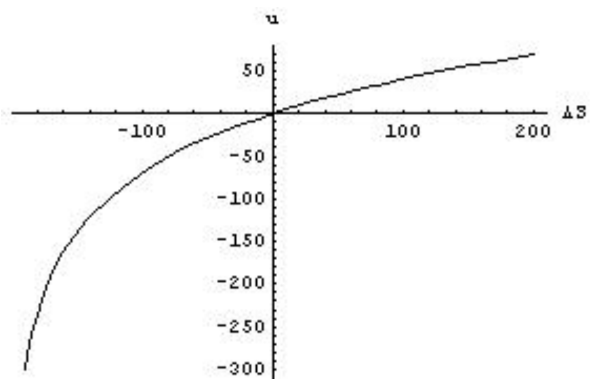
- Loss aversion is felt most strongly as financial ruin is approached:

$$\Delta S \to -S, \qquad \text{if initial wealth is } S$$

Poor Man – Rich Man

$$u = q \log \frac{S + \Delta S}{S}, \qquad \text{for } q = 100 \quad \text{(INTROSPECTION)}$$

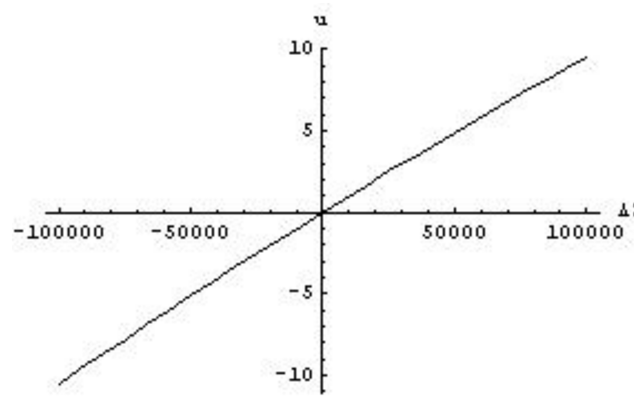
Poor man



$S_0 = 300$

$-200 \leq \Delta S \leq 200$

Rich man



$S_0 = 1.000.000$

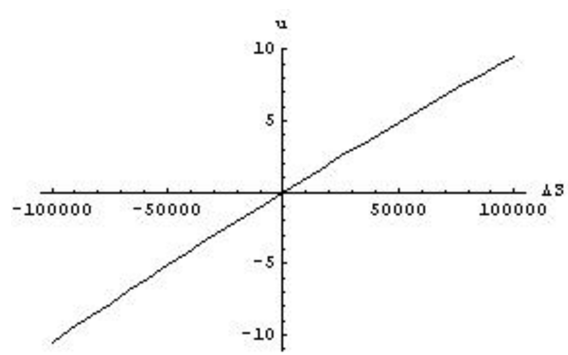
$-100.000 \leq \Delta S \leq 100.000$

Bernoulli/Weber-Fechner law

- If $S >> \Delta S$ then

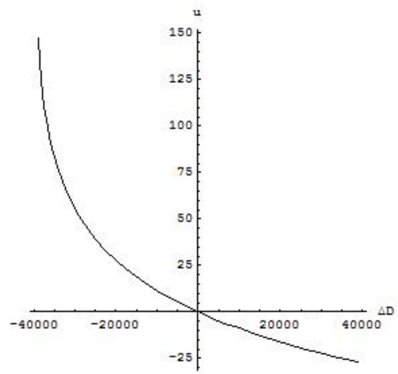
$$q \log \frac{S + \Delta S}{S} = q \log\left(1 + \frac{\Delta S}{S}\right) \rightarrow q \frac{\Delta S}{S}$$

- Linearity utility function rich man:

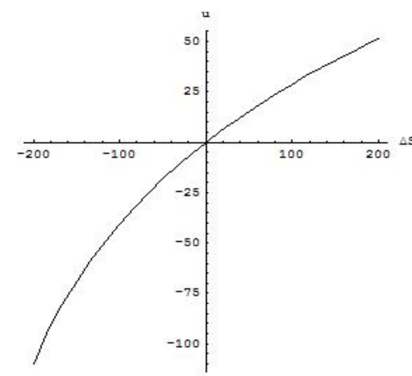


Corollary: the negative Bernoulli law for the utility of debt

Initial debt of $D = 40.000$



Initial wealth of $S = 300$



- For actual income we have **loss aversion.**
- For debt we have **debt relief**: the relief of having one's debt acquitted.
- The New Road to Serfdom; An Illustrated Guide to the Coming Real Estate Collapse, (Hudson, 2006).

Bayesian Decision Algorithm

- Construct outcome probability distributions (decision theoretical book-keeping phase).

- Construct utility probability distributions by assigning utilities to monetary outcomes, by way of Bernoulli law.

- Set-up decision theoretical inequalities, in terms of lower- or upper-bounds utility probability distribution.

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

First Case Study

The rationale of investing in flood defenses.

The Case

- Investment:

$$I = \text{investment costs associated with improvements in flood defenses}$$

- What is the maximal investment I we are willing to make to improve our flood defenses?

Investing in Flood Defenses

- Decisions:

D_1 = keep status quo

D_2 = improve the flood defenses

- Outcomes

O_1 = regular river flooding

O_2 = catostrophic river flooding

O_3 = no flooding

- Costs of outcomes

$C_1 = 10$ million euro

$C_2 = 5$ billion euro

$C_3 = 0$ euro

Simple outcome distribution, but Bayes can model more complex outcome situations

- Probabilities of outcomes under different decisions:

$$P(O_1 \mid D_1) = 10^{-2}$$
$$P(O_2 \mid D_1) = 10^{-5}$$
$$P(O_3 \mid D_1) = 1 - P(O_1 \mid D_1) - P(O_2 \mid D_1)$$

D_1 = keep status quo

D_2 = improve the flood defenses

$$P(O_1 \mid D_2) = 10^{-3}$$
$$P(O_2 \mid D_2) = 10^{-7}$$
$$P(O_3 \mid D_2) = 1 - P(O_1 \mid D_2) - P(O_2 \mid D_2)$$

Negative Weber-Fechner law: Debt Increments

- Initial wealth: $M = 10$ billion euro

- Utility function of monies: $u = q \log \dfrac{M + \Delta M}{M}$

- Unknown q falls away in decision theoretical (in)equalites. So, we may set ***q = 1***.

Utility Probability Distributions

$$p(u \mid D_1) = \begin{cases} P(O_1 \mid D_1), & u = -\log \dfrac{M + C_1}{M} \\ P(O_2 \mid D_1), & u = -\log \dfrac{M + C_2}{M} \\ P(O_3 \mid D_1), & u = -\log \dfrac{M + C_3}{M} \end{cases}$$

$$p(u \mid I, D_2) = \begin{cases} P(O_1 \mid D_2), & u = -\log \dfrac{M + C_1 + I}{M} \\ P(O_2 \mid D_2), & u = -\log \dfrac{M + C_2 + I}{M} \\ P(O_3 \mid D_2), & u = -\log \dfrac{M + C_3 + I}{M} \end{cases}$$

Risk Aversive Criterion of Action

mitigate potential losses → lower bounds action criterion

$$E(u \mid I, D_2) - k \ std(u \mid I, D_2) > E(u \mid D_1) - k \ std(u \mid D_1)$$

where

$$D_1 = \text{keep status quo}$$
$$D_2 = \text{invest in additional flood defenses}$$

and k is the sigma level of security.

Risk Aversive Criterion of Action

Investment Space: lower bound under additional flood defenses under no additional costs **minus** lower bound under status quo:

$$\left[E\left(u \mid \underline{I=0}, D_2\right) - k\ std\left(u \mid \underline{I=0}, D_2\right)\right] - \left[E\left(u \mid D_1\right) - k\ std\left(u \mid D_1\right)\right]$$

where

D_1 = keep status quo

D_2 = invest in additional flood defenses

and k is the sigma level of security.

Chebyshev's Inequality

Coverage *k*-sigma confidence interval:

$$\text{coverage} = \frac{k^2 - 1}{k^2}$$

Results Have Intuitive Orders of Magnitude

Sigma level ***k***	**coverage CI**	**max. investment** ***I***
0	n.a.	0.2×10^6
1	0	19.9×10^6
2	3/4	39.5×10^6
3	8/9	59.1×10^6
4	15/16	78.7×10^6
5	24/25	98.1×10^6
6	35/36	117.6×10^6

$k = 0$ corresponds with expected utility theory solution

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

Second Case Study

The rationale of insurance, Part I

The Case

n = contingencies covered

L = monetary damage of each contingency

p = probability of a contingency

M = wealth insurer

m = wealth costumer

P = PREMIUM ON INSURENCE

Assumption : $P\left(C_i, C_j\right) = P\left(C_i\right) P\left(C_j\right)$

Insurer: Defensive Profit Maker

D_1 = provide insurance

$$p(u \mid D_1) = \sum_{i=1}^{n} \log\left(\frac{M + P - iL}{M}\right) \binom{n}{i} p^i (1-p)^{n-i}$$

$$\rightarrow \sum_{i=1}^{n} \frac{P - iL}{M} \binom{n}{i} p^i (1-p)^{n-i}$$

D_2 = do not provide insurence

$$p(u \mid D_2) = \sum_{i=1}^{n} \log\left(\frac{M}{M}\right) \binom{n}{i} p^i (1-p)^{n-i} = \delta(u-0)$$

$$E(u \mid D_1) - std(u \mid D_1) > 0 - 0 = 0$$

Premium Lower Bound: $P > E(iL) + std(iL)$

Customer: Defensive 'Investor'

$d_1 = \text{buy insurance}$

$$p(u \mid d_1) = \sum_{i=1}^{n} \log\left(\frac{m-P}{m}\right)\binom{n}{i} p^i (1-p)^{n-i}$$

$d_2 = \text{do not buy insurence}$

$$p(u \mid d_2) = \sum_{i=1}^{n} \log\left(\frac{m-iL}{m}\right)\binom{n}{i} p^i (1-p)^{n-i}$$

$$E(u \mid d_1) - std(u \mid d_1) > E(u \mid d_2) - std(u \mid d_2)$$

Premium Upper Bound: $P < E(iL) + g\ std(iL)$

where $g \to \sqrt{1 + \gamma\ std(iL) + \frac{1}{4\ \mathrm{var}(iL)} \frac{\mathrm{var}[\mathrm{var}(iL)]}{m^2}}$

Margin of Profit (MoP) Insurer

$$E(iL) + std(iL) < P < E(iL) + g\ std(iL)$$

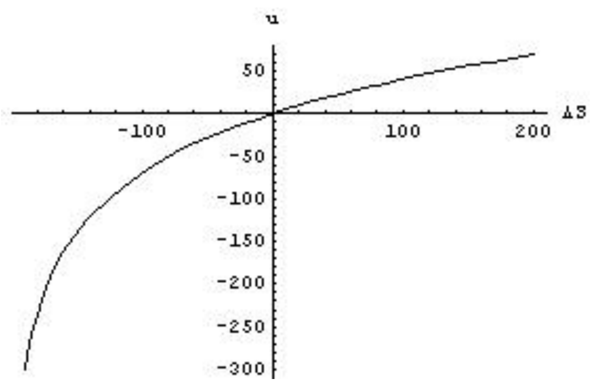
$$0 < MoP < (g-1)\ std(iL)$$

$$g \to \sqrt{1 + \gamma\ std(iL) + \frac{1}{4\ \mathrm{var}(iL)} \frac{\mathrm{var}[\mathrm{var}(iL)]}{m^2}}$$

Poor Man – Rich Man

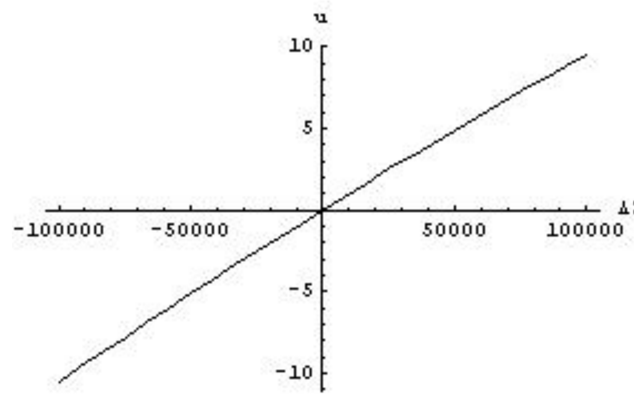
$$g \to \sqrt{1 + \gamma \; std(iL) + \frac{1}{4 \; \mathrm{var}(iL)} \frac{\mathrm{var}[\mathrm{var}(iL)]}{m^2}}$$

Poor man



$$g >> 1$$

Rich man



$$g \to 1$$

$$MOP < (g-1) \; std(iL) \to 0$$

Implication

- People need not overestimate probabilities of contingencies to want to buy insurance.

- Kahneman and Tversky are wrong on this issue.

- The concavity of their utility function + risk represented in $std(iL)$ is enough motivation

N Customers for Insurance Company

- Concavity Weber-Fechner law: g
- Risk represented by spread monetary damage $std(iL)$
- Law of large Numbers: N

$$MoP < g\,N\,std(iL) - \sqrt{N}\,std(iL)$$

$$= \left(g\sqrt{N} - 1\right)\sqrt{N}\,std(iL)$$

intuitive orders of magnitude.

- n = 10, L = 50.000, p = 0.0001, N = 10.000

- in. wealth cust. m = 1.000.000

  MoP = 40 MoP-N = 16.000.000

- in. wealth cust. m = 100.000

  MoP = 624 MoP-N = 22.000.000

Implication

- Insurance companies may take sigma-levels greater than 1 and still make a good profit.

- Concavity utility customer + spread risk + law large numbers **equals** profit.

- Neglecting second moment utility distribution leads to unawareness of effect of risk $std(iL)$ on margin of profit

- Kahneman and Tversky neglect second moment utility distribution. They are wrong in doing this.

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

The rationale of insurance, Part II

The Case

- Bottomry
  - 16th century insurance/finance construction.
  - A loan is taken out, which is only to be repaid if the vessel or merchandise arrives safely at the port of destination.
  - The premium paid for bottomry can amount to as much as 30 to 70 percent of the value of the loan

The Insurer

- Initial wealth of insurer: M

- Cost of loss of cargo: L

- Interest factor: c

- Probability cargo lost on sea: p

Insurer: Potential Wealth

$$D_1 = \text{provide bottomry contract}$$

$$M \mid D_1 = \begin{cases} M - L, & p \\ M - L + (1 + c)L, & 1 - p \end{cases}$$

$$D_2 = \text{do not provide bottomry contract}$$

$$M \mid D_2 = \left\{ M, \quad p = 1 \right.$$

Insurer: Utility pdf's

$D_1 = \text{provide bottomry contract}$

$$p(u \mid D_1) = \begin{cases} p, & u = \log \dfrac{M - L}{M} \\ 1 - p, & u = \log \dfrac{M + cL}{M} \end{cases}$$

$D_2 = \text{do not provide contract}$

$$p(u \mid D_2) = \left\{ 1, \quad u = \log \frac{M}{M} = 0 \right.$$

$$\text{Defensive profit making:} \qquad E(u \mid D_1) - std(u \mid D_1) > 0$$

Solve for Interest Factor *c*

$$c > \frac{p + \sqrt{p(1-p)}}{(1-p) - \sqrt{p(1-p)}}$$

- Risk aversive criterion of action insurer results in: *c* = **adjusted odds-ratio**.
- The insurer is effectively placing a bet by providing a bottomry contract:
- Minimally (1 + *c*) *L* must be paid by merchant for his loan *L*, which would have been forgiven had his cargo been lost

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

The Merchant

- Initial wealth merchant: m

- Cost of loss of cargo: L

- Potential profit multiplier: C

- Probability cargo lost on sea: p

Merchant: Potential Wealth

- Decision d_1 : take out loan

$$m \mid d_1 = \begin{cases} m - L + L, & p \\ m - L + L + (C - c)L, & 1 - p \end{cases}$$

- Decision d_2 : do not take out loan

$$m \mid D_2 = \begin{cases} m - L, & p \\ m + CL, & 1 - p \end{cases}$$

Merchant: Utility pdf's

$D_1 = \text{take out loan}$

$$p(u \mid d_1) = \begin{cases} p, & u = \log \frac{m}{m} = 0 \\ 1 - p, & u = \log \frac{m + (C - c)L}{m} \end{cases}$$

$d_2 = \text{do not take out loan}$

$$p(u \mid d_2) = \begin{cases} p, & u = q \log \frac{m - L}{m} \\ m + CL, & u = q \log \frac{m + CL}{m} \end{cases}$$

$$E(u \mid d_1) - std(u \mid d_1) > E(u \mid d_2) - std(u \mid d_2)$$

Solve for Interest Factor *c*

$$c < \left(C - 1 + \frac{m}{L}\right)\left[1 - \left(\frac{m - L}{m}\right)^{\frac{p + \sqrt{p(1-p)}}{(1-p) - \sqrt{p(1-p)}}}\right]$$

- The risk aversive criterion of action for the merchant results in an interest factor *c* which factors in the adjusted odds-ratio.

- Interest upper bound *c* is linear in *C* (slope = 0.078)

Summarize

- Insurer: lower bound interest factor *c*

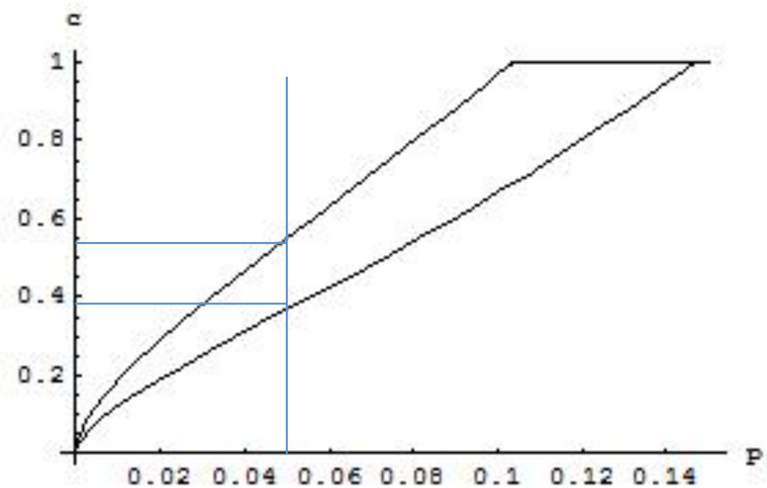
$$c > \frac{p + \sqrt{p(1-p)}}{(1-p) - \sqrt{p(1-p)}}$$

- Merchant: upper bound interest factor

$$c < \left(C - 1 + \frac{m}{L}\right)\left[1 - \left(\frac{m-L}{m}\right)^{\frac{p + \sqrt{p(1-p)}}{(1-p) - \sqrt{p(1-p)}}}\right]$$

intuitive orders of magnitude.

- Wealth merchant: $m = 1000$ guilders
- Value cargo: $L = 200$ guilders   Return factor: $C = 2$
- Interest upper bound c is linear in C (slope = 0.078)
- Historical: 30 to 70 percent of the value of the loan.



As the Merchant Gets Richer

- As initial wealth merchant *m* vastly exceeds value of cargo *L*:

$$\left(C-1+\frac{m}{L}\right)\left[1-\left(\frac{L-m}{m}\right)^{\frac{p+\sqrt{p(1-p)}}{(1-p)-\sqrt{p(1-p)}}}\right] \quad \rightarrow \quad \frac{p+\sqrt{p(1-p)}}{(1-p)-\sqrt{p(1-p)}}$$

- The margin of profit evaporates for the insurer.

- This would seem to be fundamental property for all insurance problems.

Break-down Lower Bound Insurer

- As probability of a shipwreck is $p \geq 0.5$ the lower bound of the interest factor for the insurer breaks down:

$$\frac{p + \sqrt{p\left(1-p\right)}}{\left(1-p\right) - \sqrt{p\left(1-p\right)}}$$

Risk Seeking Criterion of Action

- Indicative of impossibility of risk-aversive profit-seeking (e.g. **maximizing lower bounds** utility probability distributions).

- But investments can still be had, as for example in the first merchant expeditions to find the spice sea routes .

- 16th century spice trade had initial potential return factors of $C = 50$.

- If the return factor C is large enough we transition from risk-aversive to risk-seeking profit-seeking (e.g. **maximizing upper bounds** utility probability distributions).

Skewness Corrected Confidence Interval

Skewness Corrected Confidence Interval

- An important statistical discovery in its own right.
- Correcting for skewness in our CI-bounds allows for a richer structure in the Bayesian decision theory.
- By taking the effect of skewness on our CI-bounds into account, we come to better informed decisions.

Skewness Corrected CI-bounds

- Sigma Confidence Interval: $(\mu-\sigma, \mu+\sigma)$

- Skewness Confidence Interval for **positive skewness**:

$$\left[\mu - \frac{\sigma}{1+\dfrac{\sqrt[3]{\gamma}}{1+\gamma+\dfrac{1}{1+\gamma}}}, \mu + \left(1+\frac{\sqrt[3]{\gamma}}{1+\gamma+\dfrac{1}{1+\gamma}}\right)\sigma\right]$$

- Skewness Confidence Interval for **negative skewness**:

$$\left[\mu - \left(1-\frac{\sqrt[3]{\gamma}}{1-\gamma+\dfrac{1}{1-\gamma}}\right)\sigma,\ \ \mu + \frac{\sigma}{1-\dfrac{\sqrt[3]{\gamma}}{1-\gamma+\dfrac{1}{1-\gamma}}}\right]$$

Excellent Coverage Skewness-CI

- Normal distribution coverage is 0.68 (by construction)
- Exponential distribution coverage is 0.67 (compare with coverage of 0.68 for normal distribution)
- Beta distribution, r = 1 and n = 10, coverage is 0.63 (as $n \rightarrow \infty$ coverage goes to 0.67)
- Chi-square, r = 2, coverage is 0.67
- Gamma, α = 2 and ϑ = 1, coverage is 0.65 (as $\alpha \rightarrow \infty$ coverage goes to 0.67)
- Comparable results for 1.96 'sigma' intervals

Fact Sheet Research on Bayesian Decision Theory

- Outline of the finding of the skewness interval.
- Description of the severity of the test that this skewness interval poses for the Bayesian decision theory (when re-analyzing results of case studies).
- A test which the Bayesian decision theory passes with flying colors.

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

Certainty Bets

1. A certain gain vs. a possible greater gain or no gain.

2. A certain loss vs. a possible greater loss or no loss.

3. What is the fair probability for the uncertainty choice?
   - fair means that we are undecided between the certainty and uncertainty choices.

Positive Certainty Bets

- Let p be the probability of the uncertainty bet.
- Let O_u be the positive uncertainty outcome

  also possibility of a zero gain, with probability $1-p$

- Let O_c be the smaller positive certainty outcome.
- Let $u(O_c)$ be the utility of the positive certainty outcome

$$\text{gain} = UB_{util.pdf}(p, O_u) - u(O_c) \qquad \text{loss} = u(O_c) - LB_{util.pdf}(p, O_u)$$

Solve for ***p*** : $$UB_{util.pdf}(p, O_u) - u(O_c) = u(O_c) - LB_{util.pdf}(p, O_u)$$

Fact Sheet Research on Bayesian Decision Theory

- Most preferences for certainty bets replicated by Bayesian decision algorithm. Effect of initial wealth is taken into account (which is not done by Kahneman and Tversky)

- For a discussion of a pathological preference see **Discussion** section of the Fact Sheet.

- This pathological preference was our tipping point, where we came to trust our decision algorithm more than our intuition.

- In cases were a clear plausibility resolution is lacking, a Bayesian will trust his probability theory to provide this resolution (Jaynes' **thinking computer** [2003]).

- In the case were a clear decision resolution was lacking, we came to trust our decision theory to provide this resolution (the **decision making computer**).

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

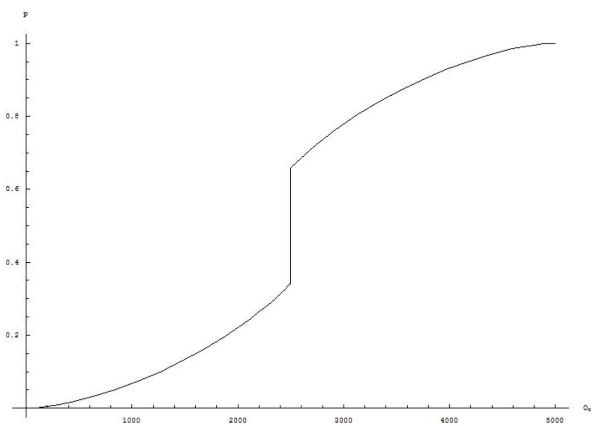
4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

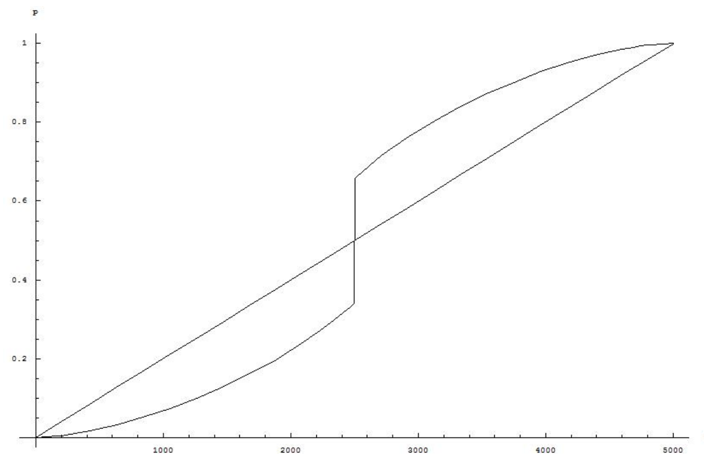
5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

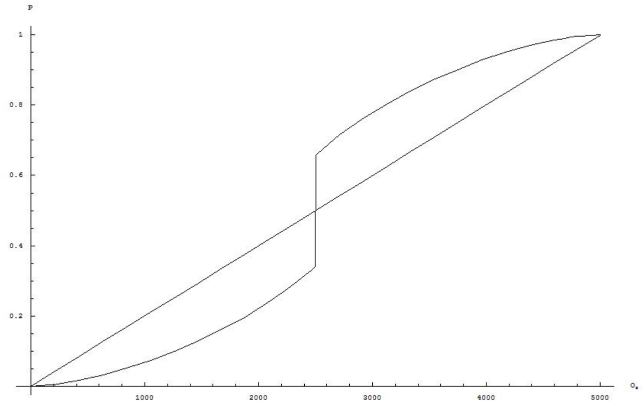
Fair Probability Functions for Linear Utilities



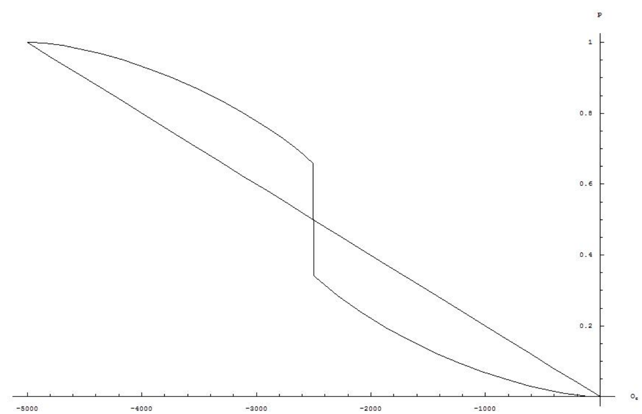
skewness-CI fair probability,
as a function of certain outcome,
given a possible uncertain gain of
5000



skewness-CI fair probability
+
sigma-CI fair probability

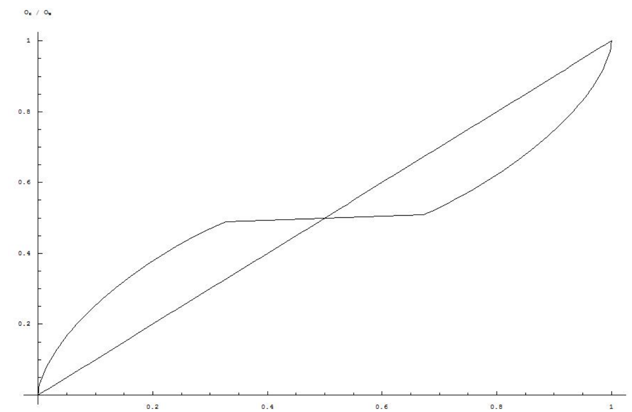


fair probability as a function of certain outcome, given a possible uncertain gain of 5000

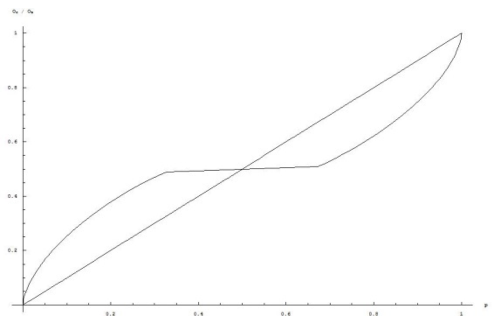


fair probability as a function of certain outcome, given a possible uncertain loss of -5000

→



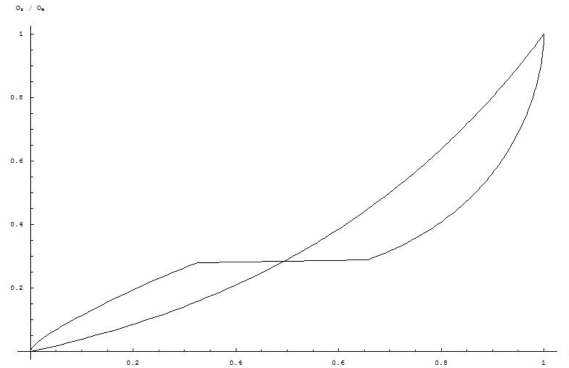
ratio **O_c / O_u** as a function of the fair probability ***p***



Linear utility
Pos. + Neg. outcomes

M >> 5000

Positive and negative outcome bets are symmetrical.

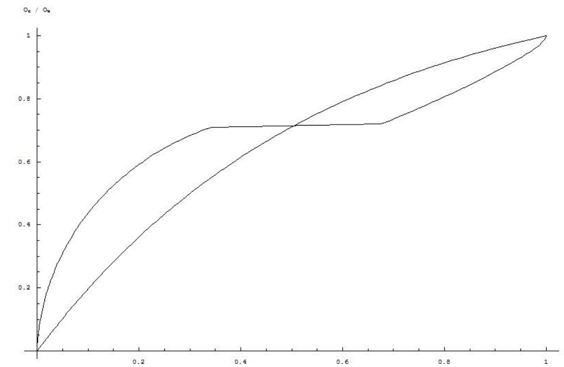


Non-linear utility
Positive outcomes

M = 1000

Low outcome ratios imply great uncertainty gain.

For low outcome ratios we are relatively quick to accept uncertainty bet.

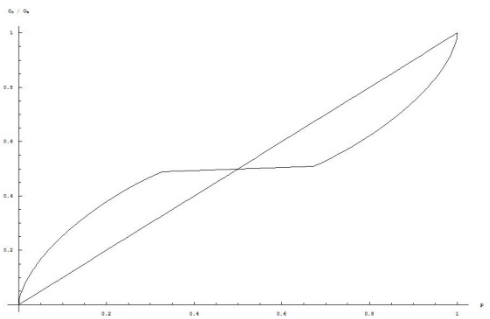


Non-linear utility
Negative outcomes

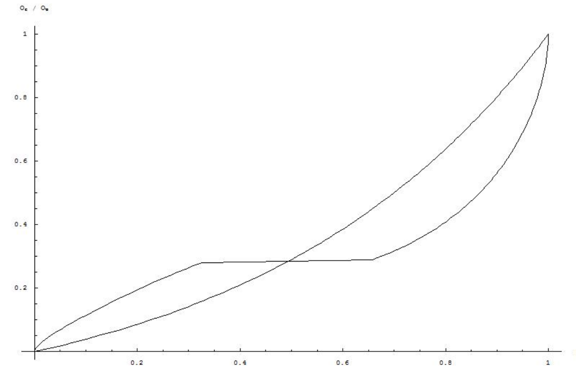
M = 6000

Low outcome ratios imply great uncertainty loss.

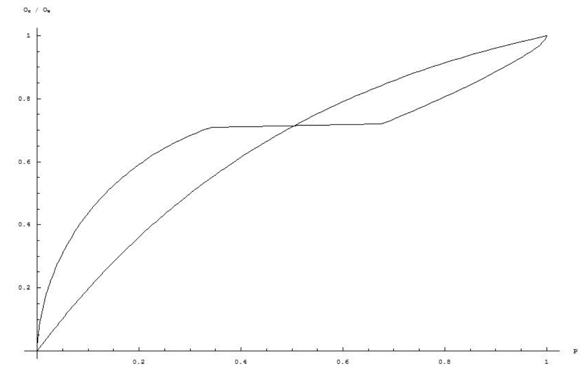
For low outcome ratios we are relatively slow to accept uncertainty bet.



Linear utility



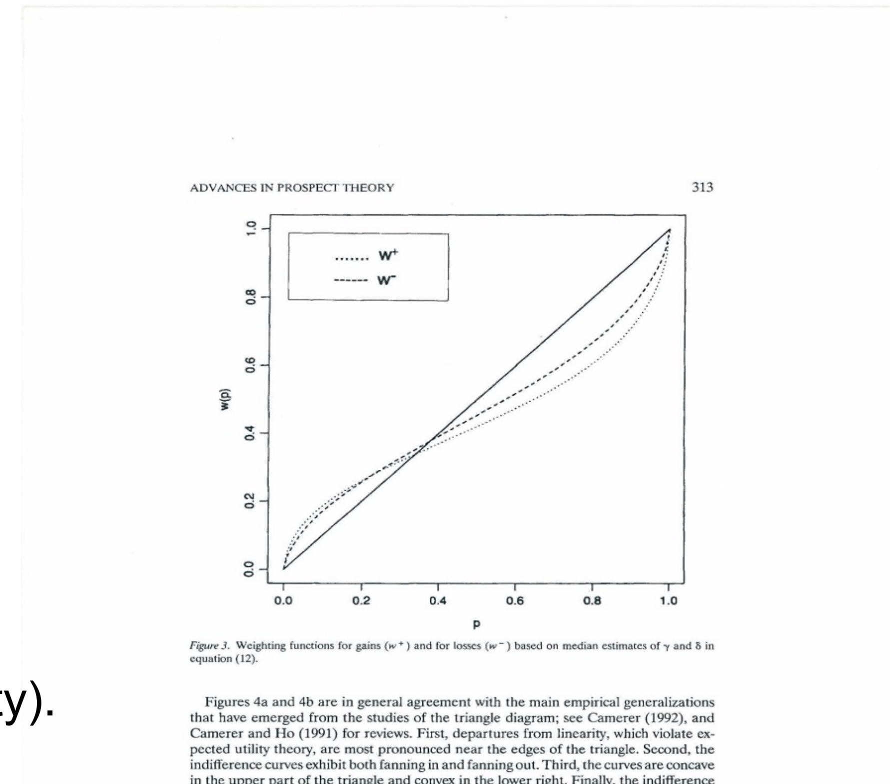
Non-linear utility
Positive outcomes



Non-linear utility
Negative outcomes

Kahneman and Tversky's probability weighing, a corner stone of their prospect theory, corroborates the intuitive relevance of the skewness corrected CI's

We derive from first principles what Kahneman and Tversky have empirically observed in their artificial betting experiments (artificiality tends to linear utility).



ADVANCES IN PROSPECT THEORY 313

Figure 3. Weighting functions for gains (w^+) and for losses (w^-) based on median estimates of γ and δ in equation (12).

Figures 4a and 4b are in general agreement with the main empirical generalizations that have emerged from the studies of the triangle diagram; see Camerer (1992), and Camerer and Ho (1991) for reviews. First, departures from linearity, which violate expected utility theory, are most pronounced near the edges of the triangle. Second, the indifference curves exhibit both fanning in and fanning out. Third, the curves are concave in the upper part of the triangle and convex in the lower right. Finally, the indifference

Non-Redundant Supporting Contacts in Non-Redundant Case Studies

1. Results have intuitive orders of magnitude.

2. If black-box is opened, inner machinery of Bayesian algorithm is intuitive.

3. Non-expected, but nonetheless extremely intuitive, interest factor resulting from Bottomry case study.

4. Bayesian decision algorithm remains stable under the most severe of tests. And was two steps ahead of our own intuition.

   Severeness of these tests is due to the skewness corrected CI-bounds.

5. Empirical data on certainty bets by Kahneman and Tversky is replicated. Moreover, Bayesian decision algorithm seems to outperform respondents.

6. Empirically observed probability weighting functions of Kahneman and Tversky are replicated by the Bayesian decision theory from first principles.

Fact Sheet Research on Bayesian Decision Theory

(van Erp, Linger, & van Gelder, 2014)

First draft:

– polemics towards Kahneman and Tversky's will be polished in future drafts (polemics need excessive attention for detail, lest one misrepresent).

– Some infidelities need to be addressed
  1. discussion Bayesian toy problem (needs explicit conditionalizing on back ground information of Fred)
  2. discussion graphs of outcome ratios as a function of the fair bets (highly non-intuitive graphs, but we had to accommodate Kahneman and Tversky)