Several Approaches to Belief Revision

Zhiguang Zhao

28/04/2014

Belief Revision

- belief revision: a transition between different states of affairs
- mechanism is not straightforward: different strength of beliefs, unreliable source, deduction ability, etc.
- central question: what constitutes a valid belief revision

Some Existing Approaches

- functional approach
- B-structures approach
- relational approach

The Functional Approach: AGM As An Example

- Alchourrón, Gärdenfors and Makinson: *On the Logic of Theory Change: Partial Meet Contraction and Revision Functions*
- AGM postulates: axiom schemes that determine whether the representation is a belief revision process
- brief idea: a body of beliefs should undergo minimal changes to accommodate new information
- dominating paradigm for reasoning about belief revision

The Functional Approach: AGM Postulates

$*$ is a belief revision function which takes a deductively closed set K of beliefs and formula φ to the deductively closed set $K * \varphi$ of beliefs.

- A1: $K * \varphi$ is deductively closed
- A2: $\varphi \in K * \varphi$
- A3: $K * \varphi \subseteq Cn(K \cup \{\varphi\})$
- A4: if $\neg\varphi \notin K$ then $Cn(K \cup \{\varphi\}) \subseteq K * \varphi$
- A5: $K * \varphi = Cn(\{\bot\})$ if and only if $\vdash \neg\varphi$
- A6: if $\vdash \varphi \leftrightarrow \psi$ then $K * \varphi = K * \psi$
- A7: $K * (\varphi \wedge \psi) \subseteq Cn((K * \varphi) \cup \{\psi\})$
- A8: if $\neg\psi \notin K * \varphi$ then $Cn((K * \varphi) \cup \{\psi\}) \subseteq K * (\varphi \wedge \psi)$

The Functional Approach: Some Problems about AGM

- A1: must be deductively closed?
- A2: the agent can choose not to believe the new information
- global revision: all beliefs need to be taken into account during the revision
- determination: only one result of the revision

The B(elief)-Structures Approach

- Chopra and Parikh: Relevance Sensitive Belief Structures
- brief idea: only relevant beliefs should be affected by the new coming information
- language splitting model: the base model of B-structure for splitting the language into different "subjects"
- subject matter: a measure of the information content of a proposition to test the relevance between two propositions
- B-Structure: the model for belief revision

The B-Structures Approach: Language Splitting Model

Let $L = (L_1, L_2, \ldots, L_n)$ be a family of (mutually disjoint) subsets of L, and let T be a theory of the language L. Then $(L_1, L_2, \ldots, L_n)$ split L relative to T if and only if for each i in $1, \ldots, n$ there exists $\varphi_i \in L_i$ such that $T = Cn(\varphi_1, \ldots, \varphi_n)$.

The B-Structures Approach: Subject Matter and Relevance

- subject matter: a way of talking about information content of a formula
- Let φ be a formula of a finite propositional language L. The subject matter of φ is the smallest language, denoted L_φ, of a formula that can be used to express φ. Two formulae φ and ψ of L are relevant to each other if $L_\varphi \cap L_\psi \neq \varnothing$.

The B-Structures Approach: Belief Structure

A belief structure (B-structure) on L is a set $B = (L_i, T_i)_{i \in I}$ where $I = \{1, \ldots, n\}$, $L = \bigcup_{i \in I} L_i$ and for each $i \in I$, T_i is a consistent, finitely axiomatisable theory in L_i. For each $i \in I$, Γ_i is a set of explicit beliefs of an agent, expressed in the language L_i, such that $T_i = Cn(\Gamma_i)$.

- Here T_i and Γ_i are called implicit and explicit beliefs, respectively.
- Each L_i need not be mutually disjoint, and the union of each T_i need not be consistent: local consistency vs global inconsistency.
- Different possibilities are mentioned in Chopra and Parikh's paper, and all of them are deterministic in the sense that there is only one outcome for the belief revision operation.

The B-Structures Approach: Option B Revision

- brief idea: theories that are affected by new information are merged
- given new information φ, first take $\Gamma_\varphi = \bigcup\{\Gamma_i \mid L_i \cap L_\varphi \neq \varnothing\}$, then replace each L_i relevant to φ by $\bigcup\{L_i \mid L_i \cap L_\varphi \neq \varnothing\}$, and then replace each corresponding T_i by $Cn(\Gamma_\varphi) * \varphi$.

The Relational Approach

- brief idea: belief revision can have different outcomes
- epistemic entrenchments: relative strength of a particular belief

The Relational Approach: Epistemic Entrenchment

Fix a consistent logic L, and take the L-theory G to be the set of beliefs held by an agent. An epistemic entrenchment for G is then a binary relation $\leq_e \subseteq \Phi \times \Phi$ such that:

- E1: if $\varphi \leq_e \psi$ and $\psi \leq_e \gamma$ then $\varphi \leq_e \gamma$
- E2: if $\varphi \vdash \psi$ then $\varphi \leq_e \psi$
- E3: if $\varphi \leq_e \psi$ and $\varphi \leq_e \gamma$ then $\varphi \leq_e \psi \wedge \gamma$
- E4: if $\bot \notin G$ then $\varphi \notin G$ if and only if $\varphi \leq_e \bot$
- E5: if $\top \leq_e \varphi$ then $\vdash \varphi$

The Relational Approach: Basic Definitions

- fallbacks: filters relative to $\leq_e$
- Given a formula φ, a fallback $H \subseteq G$ is called φ-permitting if $\neg\varphi \notin H$ and maximal φ-permitting if it is maximal w.r.t. this property.
- relational revision: given information φ, H is a revision of G if either $\neg\varphi \in L$ and $H = \Phi$ or there exists a maximal φ-permitting fallback K of G such that $H = Cn(K \cup \{\varphi\})$.

The Relational Approach: Axioms

- R1: there exists a theory H such that $H \in R_\varphi(G)$
- R2: if $H \in R_\varphi(G)$, then $\varphi \in H$
- R3: if $\neg\varphi \notin G$ and $H \in R_\varphi(G)$, then $H = Cn(G \cup \{\varphi\})$
- R4: if $\neg\varphi \notin L$ and $H \in R_\varphi(G)$, then $\bot \notin H$
- R5: if $\vdash \varphi \leftrightarrow \psi$, then $H \in R_\varphi(G)$ if and only if $H \in R_\psi(G)$
- R6: if $H \in R_\varphi(G)$ and $\neg\psi \notin H$, then $Cn(H \cup \{\psi\}) \in R_{\varphi\wedge\psi}(G)$
- R7: if $H \in R_\varphi(G)$ and for all theory K we have that if $K \in R_{\varphi\vee\psi}(G)$ then $\neg\varphi \notin K$, then there exists a theory K such that $K \in R_{\varphi\vee\psi}(G)$ and $H = Cn(K \cup \{\varphi\})$

If R_φ is a function, the the postulates above revert to AGM postulates

Summary

- Functional Approach:deterministic, global
- B-Structures Approach:deterministic, local
- Relational Approach:non-deterministic, global

Author's Approach

- both local and non-deterministic
- use ideas from sheaf theory to model belief revision in this manner